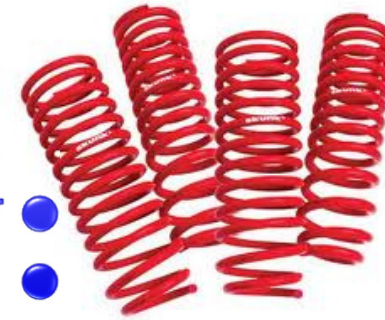


And a lot of things  
can be approximated  
as springs!



# Main Idea Today:

# Springs

Pens



Summary of Today's Class

<http://www.youtube.com/watch?v=-KqJXLJBb6s>

Shocks

# Predict

A weight is suspended by a spring. The spring is then stretched until the weight is just above the eggs. When the spring is released, the weight is pulled up by the contracting spring and then falls back down under the influence of gravity. **Ignoring air resistance**, on the way down, it

- A. reverses its direction of travel well above the eggs
- B. reverses its direction of travel precisely as it reaches the eggs
- C. makes a mess as it crashes into the eggs (yuck)



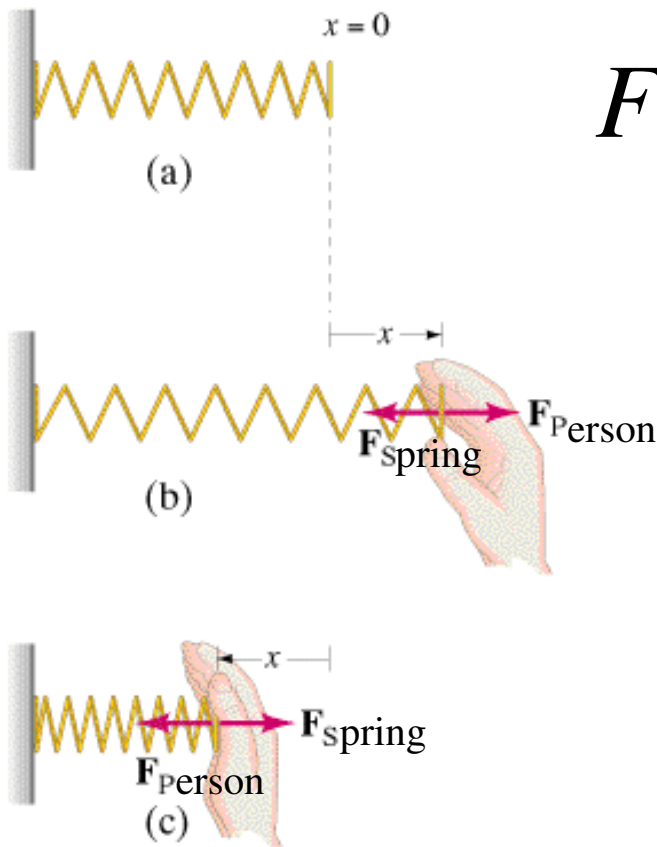
# Hooke's Law (Restoring Force)

The force a spring provides to an attached object is proportional to the amount that the spring is stretched or compressed

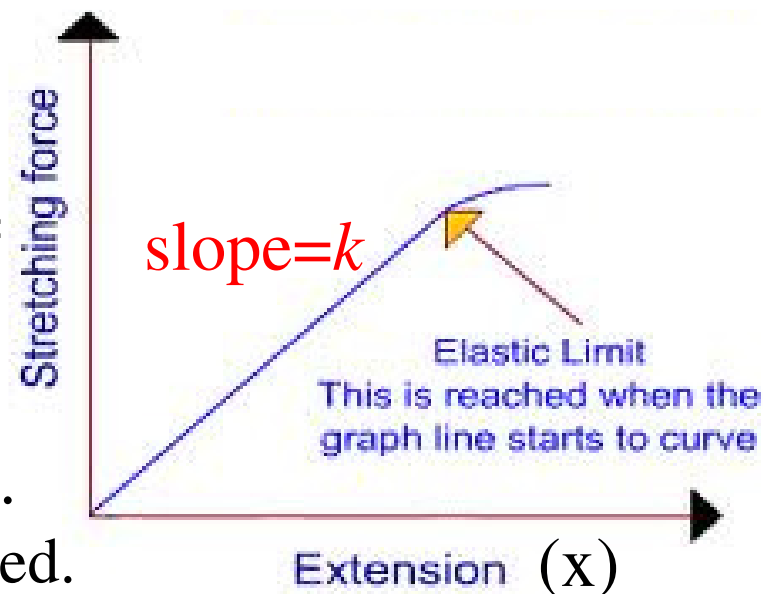
$$F_S = -kx \quad (\text{Hooke's Law})$$

$k$  is the spring constant

$x = 0$  when not pulled/pushed



$$F_s =$$



Springs that don't like to stretch have high  $k$ .

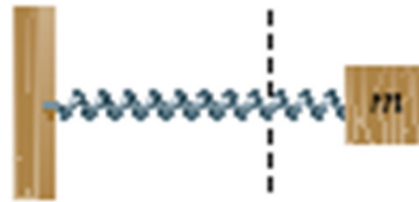
Most things act a little like springs when compressed.

**Ch. 13:**  
**Oscillatory**  
**Motion**

# One oscillation (cycle)

$$F_s = -kx = ma$$

Maximum displacement



$$F_x = F_{\max}$$

$$a = a_{\max}$$

$$v = 0$$

Equilibrium

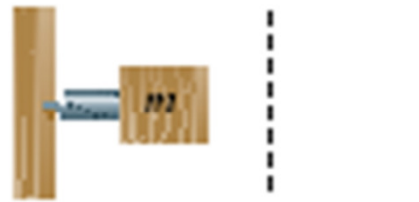


$$F_x = 0$$

$$a = 0$$

$$v = v_{\max}$$

Maximum displacement



$$F_x = F_{\max}$$

$$a = a_{\max}$$

$$v = 0$$

Equilibrium

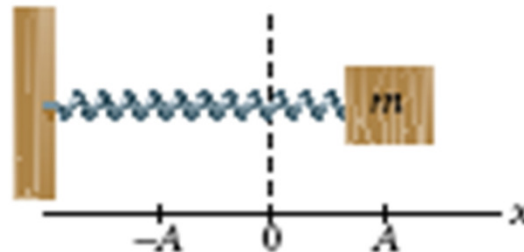


$$F_x = 0$$

$$a = 0$$

$$v = v_{\max}$$

Maximum displacement



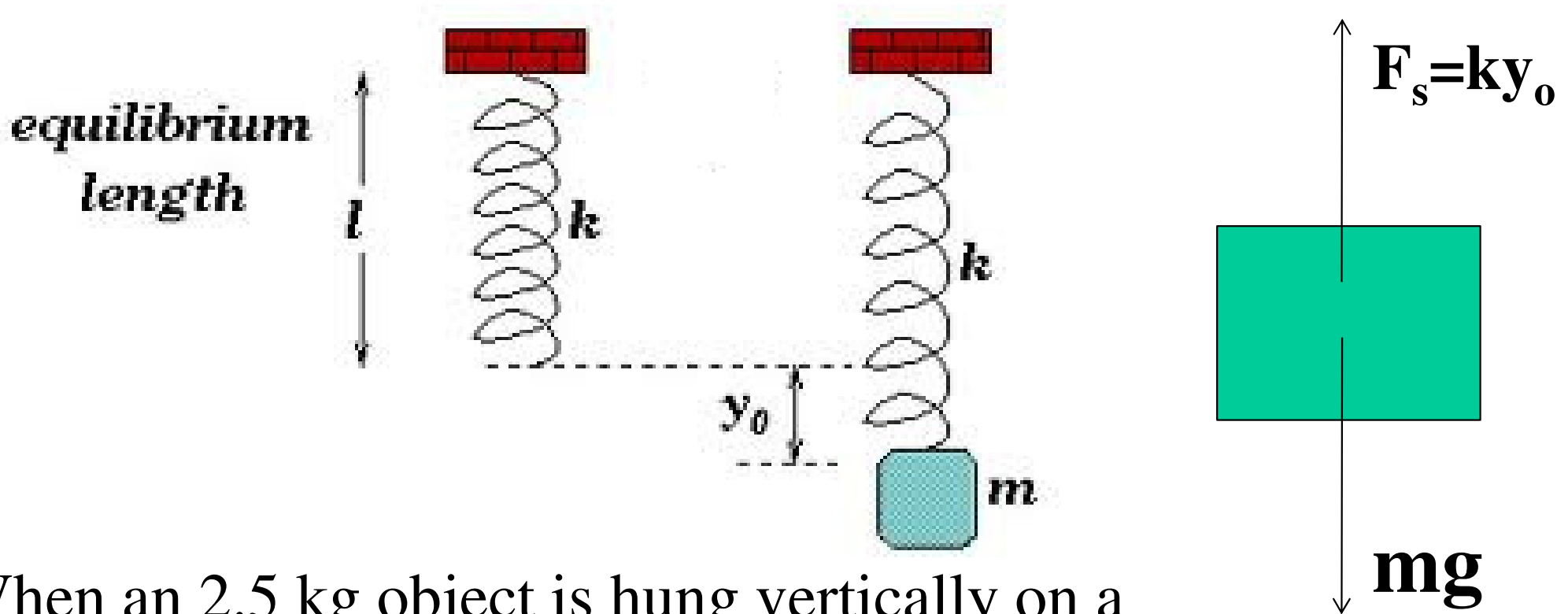
$$F_x = F_{\max}$$

$$a = a_{\max}$$

$$v = 0$$

The time to complete one cycle is called the period  $T$ .

# Vertical Springs



When an 2.5 kg object is hung vertically on a certain light spring, the spring stretches to a distance  $y_0$ . What **force** does the spring apply to the object?

- b) If the string stretches 2.76 cm from this mass, what is the force constant of the spring?
- c) What is the force if you stretch it 8 cm?

## Free Body Diagram

I might ask any of these on the test.

# Spring (Elastic) Potential Energy

## Elastic Potential Energy

When compressed or stretched, a spring gains elastic potential energy.



static



compressed



stretched

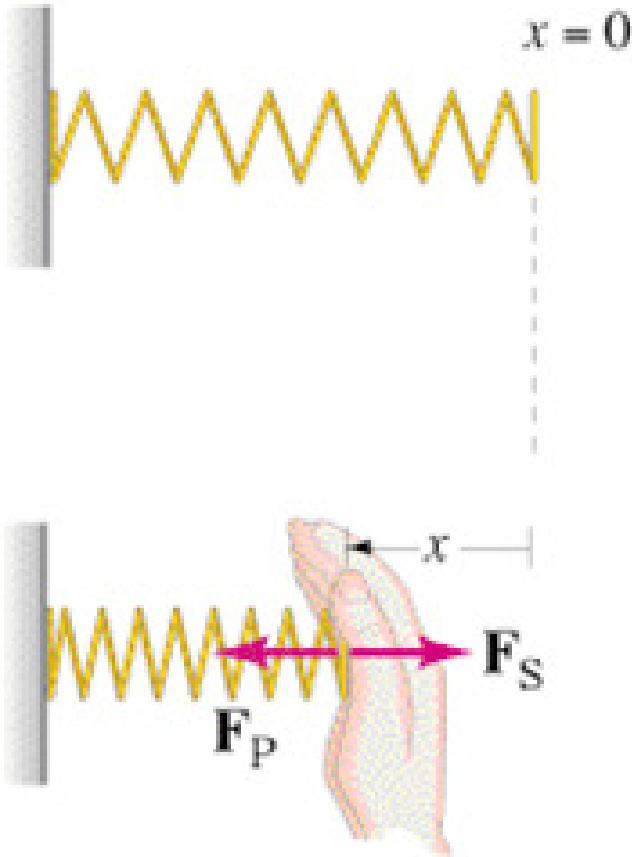
# How Do We Find Spring Potential Energy?

$$F_S = -kx \quad (\text{Hooke's Law})$$

$x = 0$  is equilibrium

$$W_C = -\Delta PE$$

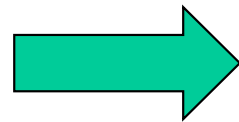
$$W = F_{avg} \Delta x$$



If spring is compressed a distance  $x$ , **average** spring force is:

$$\bar{F}_{S,average} = \frac{1}{2} (0 - kx) = -\frac{1}{2} kx$$

$$\Rightarrow W_S = \bar{F}_{avg} x = \left(-\frac{1}{2} kx\right) x = -\frac{1}{2} kx^2$$

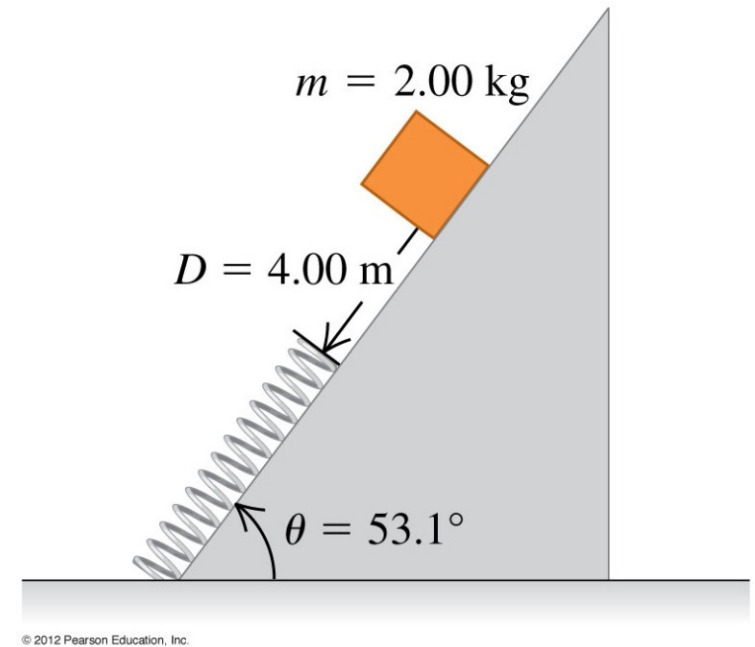


$$\Delta PE_S = PE_2 - PE_1 = -W_S = \frac{1}{2} kx^2$$

$PE_1 = 0$ , so

$$PE_S = \frac{1}{2} kx^2$$

A block is released from rest on a frictionless incline as shown. When the moving block is in contact with the spring and **compressing** it, what is happening to the gravitational potential energy  $PE_{\text{grav}}$  and the elastic potential energy  $PE_{\text{el}}$ ?



- A.  $PE_{\text{grav}}$  and  $PE_{\text{el}}$  are both increasing.
- B.  $PE_{\text{grav}}$  and  $PE_{\text{el}}$  are both decreasing.
- C.  $PE_{\text{grav}}$  is increasing;  $PE_{\text{el}}$  is decreasing.
- D.  $PE_{\text{grav}}$  is decreasing;  $PE_{\text{el}}$  is increasing.
- E. The answer depends on how the block's speed is changing.



**Q129**

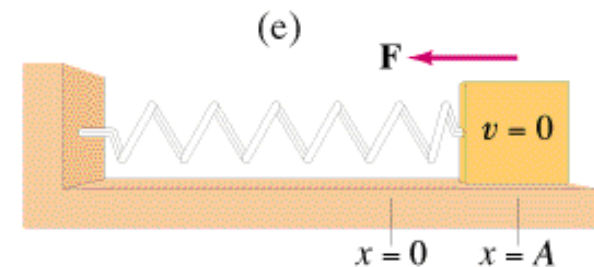
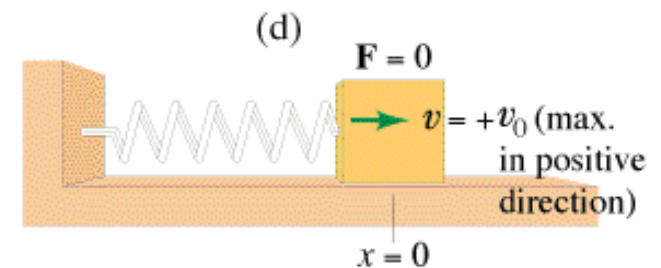
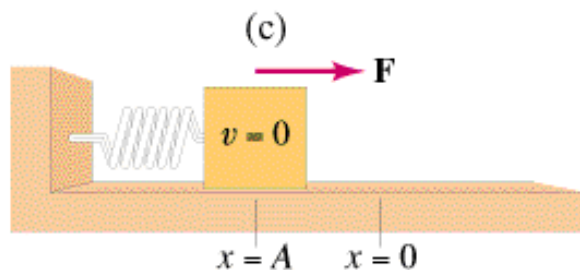
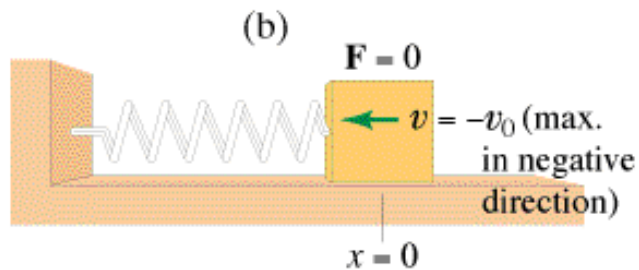
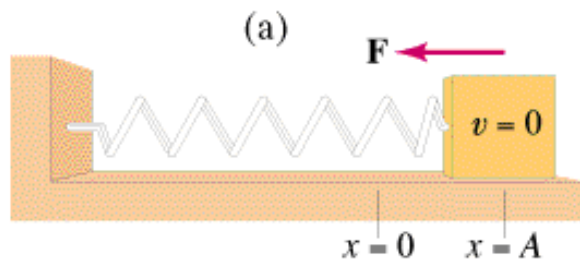


# Amplitude of Oscillation

Amplitude ( $A$ ) is maximum distance from equilibrium point

**What is the velocity at this point?**

**Only have potential energy**

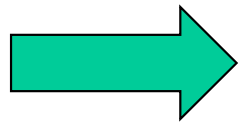


# Energy in an oscillator

- Total energy of system (no frictional forces doing work):

$$E = KE + PE = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

- $E$  is the same everywhere - as KE increases PE decreases and vice-versa
- $E$  in terms of amplitude: when  $x = A$ ,  $v = 0$



$$E = \frac{1}{2}kA^2$$

# Velocity as a Function of Position

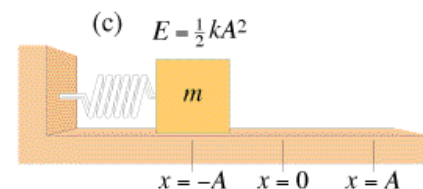
(instead of time)

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

- Conservation of Energy allows a calculation of the velocity of the object at any position

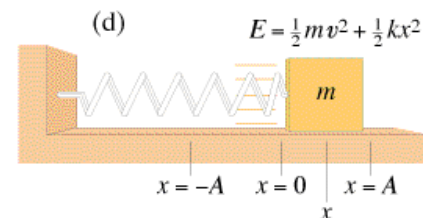
$$v = \pm \sqrt{\frac{k}{m}(A^2 - x^2)}$$

- Speed is a maximum at  $x = 0$
- Speed is zero at  $x = \pm A$
- The  $\pm$  indicates the object can be traveling in either direction

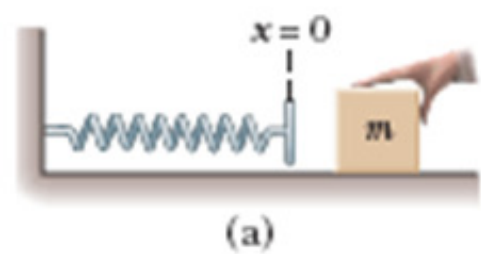


$$\frac{1}{2}mv_{\max}^2 = \frac{1}{2}kA^2$$

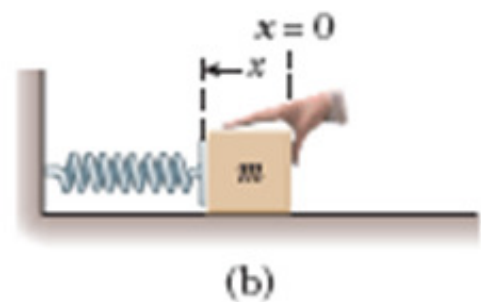
$$\Rightarrow v_{\max}^2 = \frac{k}{m}A^2$$



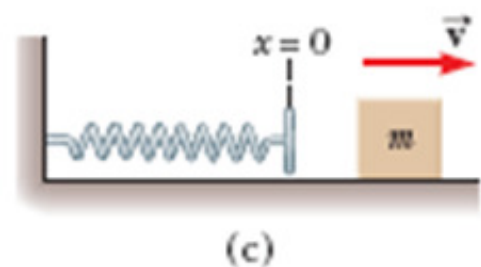
A spring with spring constant 300 N/m is attached to an object whose mass is 2.0 kg. If the spring is initially stretched  $A=0.25$  m, what is the velocity of the object at  $x = 0$ ,  $-A$  and  $A/2$ ?



$$\frac{1}{2} m v^2 + \frac{1}{2} k x^2 = \frac{1}{2} k A^2$$



$$v = \pm \sqrt{\frac{k}{m} (A^2 - x^2)}$$





# Previous Hard Test Question

A person who is still oscillating up and down **after** completing a bungee jump can be treated as a mass on a **vertical spring**, where gravity might also play a role. **In this approximation**, where is the magnitude of the **net acceleration** of the person in oscillation maximized? [Hint: Draw FBDs.]



- A. The acceleration is always  $g=9.8 \text{ m/s}^2$ .
- B. At the equilibrium length of the bungee cord.
- C. When the person is closest to the ground.
- D. When the person is farthest from the ground.
- E. Both when the person is closest and farthest from the ground.



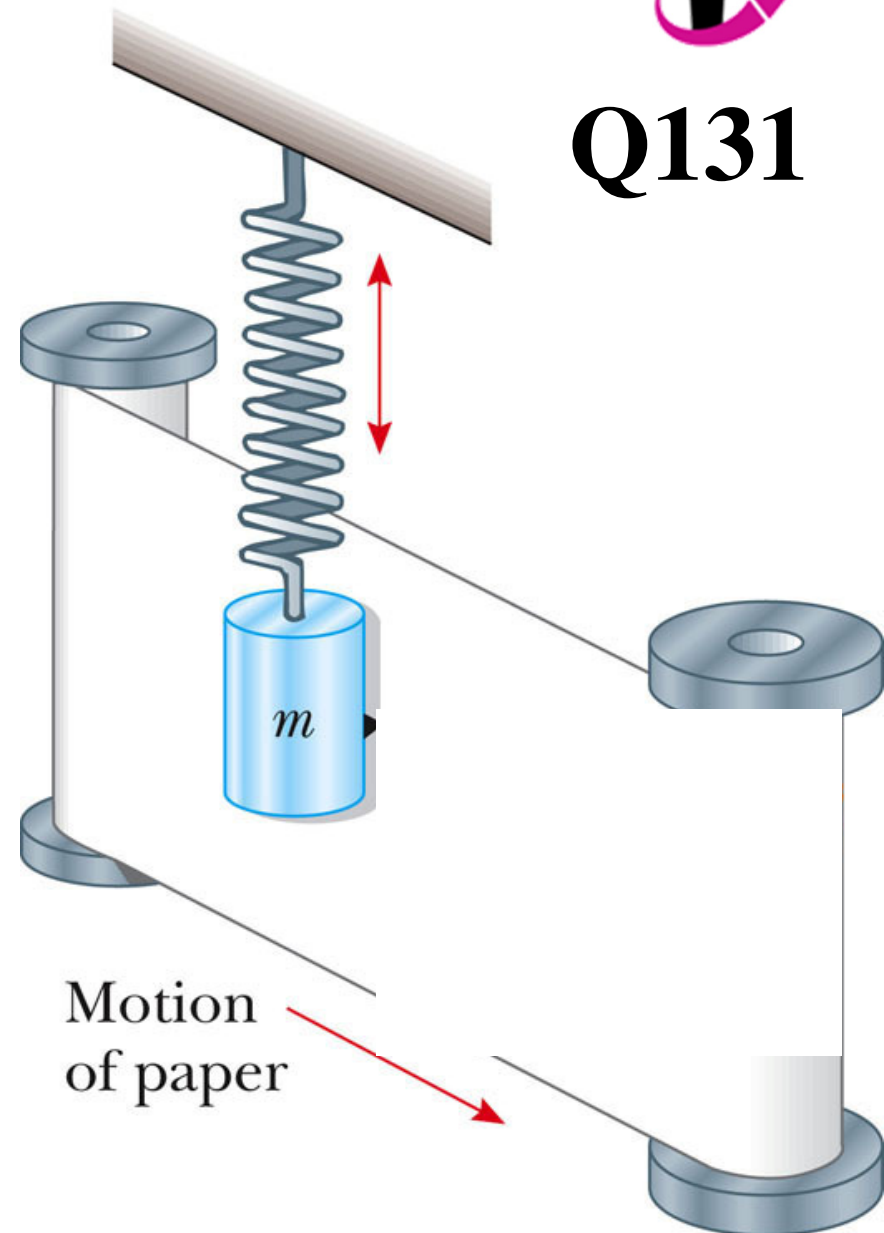
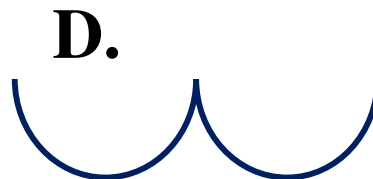
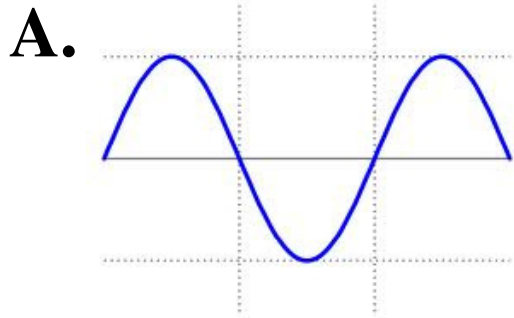
**Q130**

# Graphing the Motion of Springs



Q131

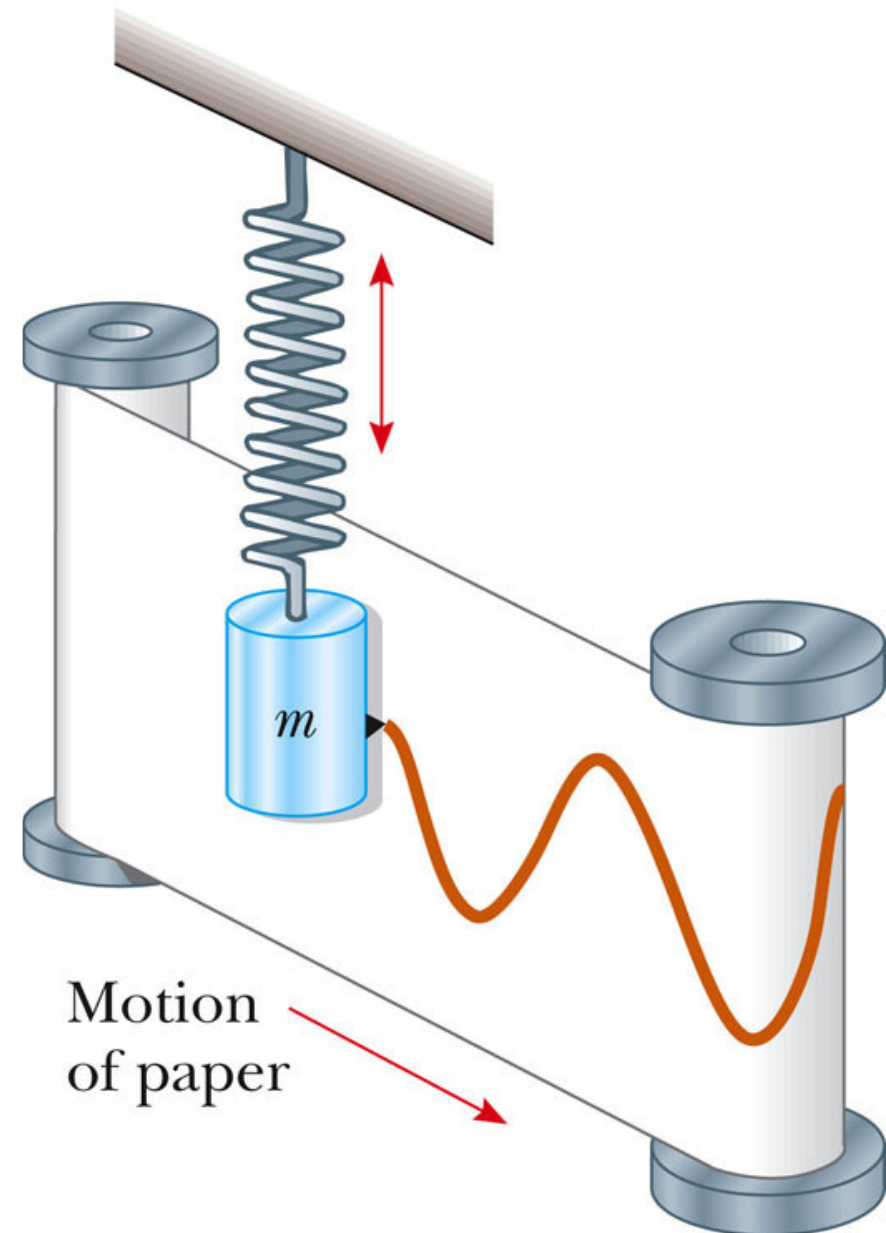
The paper moves at a constant speed underneath the pencil. If we were to graph what we observe, what would the **position versus time** graph look like?



# Graphing the Motion of Springs

The periodic motion of a spring is called sinusoidal motion, since it follows a sine or cosine relation.

This periodic motion is Simple Harmonic Motion.



# Simple Harmonic Motion

- Any vibrating system with  $F$  proportional to  $-x$  like Hooke's law ( $F=-kx$ ) undergoes SHM
- System is called a simple harmonic oscillator (SHO)
  - Ex: Spring; pendulum (for small amplitudes), a car stuck in a ditch being "rocked out", a person on a swing, vibrating strings, even sound (Ch.14)!





# Main Ideas in Class Today

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After today's class, you should be able to:

- Use Hooke's Law to find the force on an object attached to a spring based on  $\Delta x$  (or vice versa)
- Determine the potential energy in a spring and the work done by a spring
- Apply conservation of energy principles to springs

**Suggested Practice Problems:** 5.21, 5.35, C13.1, C13.3, 13.1, 13.3, 13.5, 13.9, 13.11

# Clicker Answers

1=E, 2=E, 3=D, 4=D, 5=B, 6=C, 7=A, 8=C, 9=E, 10=A,  
11=C, 12=B, 13=C, 14=E, 15=A, 16=B, 17=C, 18=B, 19=D,  
20=A, 21=B, 22=B, 23=A, 24=B, 25=A, 26=E, 27=C, 28=C,  
29=B, 30=D, 31=C, 32=B, 33=D, 35=D, 36=A, 37=B, 38=C,  
39=E, 40=B, 41=B, 42=C, 43=A, 44=A, 45=C, 46=D, 47=E,  
48=B, 49=A, 50=A, 51=C, 52=C, 53=A, 54=C, 55=E, 56=D,  
57=E, 58=D, 59=E, 60=A, 61=B, 62=B, 63=D, 64=A, 65=A,  
66=C, 67=D, 68=C, 69=B, 70=A, 71=B, 72=B, 73=A, 74=B,  
75=A, 76=B, 77=A, 78=A, 79=D, 80=A, 81=B, 82=C, 83=B,  
84=B, 85=B, 86=B, 87=C, 88=B, 89=A, 90=B, 91=D, 92=B,  
93=B, 94=B, 95=B, 96=C, 97=C, 98=2, 99=2, 100=D, 101=B,  
102=A, 103=D, 104=B, 105=B, 106=A, 107=C, 108=B,  
109=A, 110=C, 111=A, 112=B, 113=A, 114=B, 115=B,  
116=B, 117=D, 118=A, 119=C, 120=A, 121=E, 122=D,  
123=D, 124=B, 125=A, 126=D, 127=D, 128=B, 129=D,  
130=D, 131=A

(today's clicker problems)